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PROPAGATION OF COMPLEX DISCONTINUITIES WITH PIECEWISE CONSTANT AND VARIABLE VELOCITIES ALONG CURVILINEAR AND BRANCHING TRAJECTORIES*

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The method of functionally invariant solutions of wave equations utilizing the principle of superposition is used to construct exact solutions for a system of complex discontinuities propagating with piecewise constant velocities along curvilinear and branching trajectories. A passage to the limit is shown in the course of which the solution constructed yields a solution for the case of the propagation of discontinuities with variable velocities along smooth curvilinear trajectories. It is shown that if the propagation of a major discontinuity (crack) begins with the formation of a pure shear element, then as the velocity increases, dislocation components of the displacement vector will appear at the discontinuity; if on the other hand the propagation of a major crack begins with the formation of a pure dislocation element, then as the velocity of propagation of the crack increases, it will branch and considerable shear components of the displacement vector will form on the branched segments of the crack. The minimum values of the branching angles are determined. Theoretical seismograms are given computed for the curvilinear and branching cracks consisting of alternating elements with dislocation and shear components of the displacement vector at the discontinuity.

The high-frequency emissions observed in nearly all seismograms cannot be explained within the framework of the model representations developed in /1-4/ and based on the concept of a seismic focus as the only propagating shear fracture area. In this connection, considerable efforts have been made in developing the concepts of a discrete, intermittent (jump-like) growth of the fracture process at the foci of tectonic earthquakes /5-10/**. (**Bykovtsev A.S. Some problems of the dynamic theory of dislocation discontinuities and their seismological applications. Candidate Dissertation. Mosk. Gorn. Inst. 1979.) The analyis of numerical solutions of problems concerning the jump-like motions of shear cracks /5, 6/ has led to the formulation of an essentially new "barrier" model of the process of growth at the focus of a tectonic earthquake /6/ which represents a generalization of the models /1-4/. The work done in this area is generalized in the monographs /11-13/, where the investigations carried out are discussed in detail.

The use of the force approach in /14-16/ to describe the cracks, i.e. in the case when the stresses are specified on the surface of the crack, considerably limits the possibility of modelling the motion occurring in the course of fracture within simple, as well as complex foci of tectonic earthquakes. Thus the analytic solutions /14, 15/ were not used for an effective analysis of specific seismological problems, since the numerical realization of the analytic solutions and the derivation of physical conclusions from /14, 15/ presented a very complex, practically insurmountable problem requiring the solution of pentuple integrals. For this reason, many workers preferred the kinematic description of the fracture areas, and this proved to be the most productive approach in investigating the special features of the wave fields originating at simple as well as complex earthquake foci. At present, a sufficient number of examples exist of the use of the kinematic approach for effective modelling of the special features of seismic emission (/1-3/ etc.). The basic idea of constructing solutions consists of the fact that the solution of wave equations for a point source in the form of a momentless equilibrium dipole (the emission field of such a point source is equivalent to the emission field of an infinitesimal dislocation) was integrated over the fracture area of the given configuration. The solutions obtained were used to analyse a number of interesting problems, it was found, however, that generalizing the method to embrace a complex system of discrete discontinuities leads to an exponential growth in the computer time necessary to carry out such calculations.

A basically different method was developed by in /7-9, 17-23/, where the magnitude and direction of the displacement vector at the fracture was specified over the whole of the fracture area in the form of a boundary condition, and the boundary value problems of the

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theory of elasticity were then solved. This is how the problem of a jump-like propagation of a circular fracture area was solved in /7/ and the analogues of the well-known Griffith, Yoffe and Panasyuk-Lozovoi problems were solved in the kinematic formulation in /17-20/. It was shown, as a result, that the behaviour of the dislocation discontinuities is qualitatively analogous to the behaviour of cracks, the sole difference being in the values of the numerical ratios of some of the parameters, and is connected with the fact that in a small neighbourhood of the crack edge the stresses and deformations are proportional to the distance of the power $-\frac{1}{2}$, while in the theory of dislocation discontinuities the behaviour of the stresses and deformations in a small neighbourhood of the crack edge depends on the configuration of this edge /21/.

The use of the kinematic description of the discontinuities enabled the author, in the work cited in the footnote, to construct a solution, and to calculate the theoretical seismograms for a curvilinearly propagating shear fracture. The fundamentally novel, basic qualitative result which emerged from the solution obtained was that the seismic emission may become sign-alternating in certain directions. This clarified the physical nature of the sign-alternating trains of impulses observed in real seismograms even after removing the effect of the factors connected with the layered nature of the earth's crust. The result was used in /8, 9/ to offer a basically new model of the fracture processes occurring at the foci of tectonic earthquakes, based on the jump-like propagation of the major fracture along a piecewise smooth curvilinear trajectory. The model generalizes substantially the models of /1-4/, as well as the barrier model /5, 6/. A detailed analysis of the wave fields generated by the jump-like propagation of the discontinuity was carried out in /22/, together with a study of how the form of the impulses depends on the function describing the displacement vector along the discontinuity, and analysis of the form of the theoretical seismograms produced by a curvilinear unit shear crack. A solution is obtained for an antiplane system of star-like cracks, and the diagram of the directions of seismic emission is analysed in /23/.

Thus the fact that the growth of fractures in rocks occurs not over a smooth flat surfaces, but along some curved or branching surfaces, makes the problem of the dynamic growth of curvilinear cracks consisting of alternating shear and shift elements, particularly interesting. Therefore, the main object of this paper will be to study representations of dynamic fields of elastic perturbations generated by arbitrary cracks propagating with varying velocity along arbitrary curvilinear trajectories. The solutions obtained can be used to solve one of the fundamental problems of theoretical seismology, namely to determine new relations connecting the elastic displacement fields with the position, orientation, the trajectories and velocities of propagation of cracks in the earth's interior. Another application (which may prove even more important than the seismological one) is the study of acoustic emission signals in stressed constructions. These signals portend the total disintegration of the structure (they emanate from the dynamic dislocations and cracks).

1. Basic representations and construction of the fundamental solution. Let us consider the following auxiliary problem. Let a generalized dislocation discontinuity begin to propagate at the initial instant $t=0$ from the origin of the Cartesian system of coordinates $Oxyz$ with constant velocity v in the direction of the x axis, through a homogeneous isotropic medium whose shear modulus is μ. We shall regard as a dislocation discontinuity, the discontinuity whose description involves the kinematic method of specifying it, i.e. the magnitude and direction of the displacement jump is specified along the fracture line, depending on the coordinate and the time, at every point of the fracture surface. The initial conditions are zero.

We denote by U_x, U_y, U_z the components of the displacement vector along the corresponding x, y, z axes of the Cartesian rectilinear coordinates. The problem is assumed to be plane, i.e. U_x, U_y, U_z are functions of x and y only.

The basic equations of the dynamic theory of elasticity in this case have the form

$$U_x = U_x^p + U_x^s, \quad U_y = U_y^p + U_y^s, \quad U_z = U_z^s \tag{1.1}$$
$$\Delta U_\eta^p = \frac{1}{c_p^2}\frac{\partial^2 U_\eta^p}{\partial t^2} \quad (\eta = x, y); \quad \Delta U_\eta^s = \frac{1}{c_s^2}\frac{\partial^2 U_\eta^s}{\partial t^2} \quad (\eta = x, y, z)$$
$$\frac{\partial U_x^p}{\partial y} = \frac{\partial U_y^p}{\partial x}, \quad \frac{\partial U_x^s}{\partial x} = -\frac{\partial U_y^s}{\partial y}; \quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Here the superscripts p, s correspond to the longitudinal and transverse displacement components, and c_p and c_s are the velocities of the longitudinal and transverse wave respectively ($c_p > c_s$).

Let the dislocation crack be described by a homogeneous function of zero dimension $\mathbf{f}(r/t)$. The following expansion is admissible in the general case:

$$\mathbf{f}(r/t) = f_1(r/t)\,\mathbf{i} + f_2(r/t)\,\mathbf{j} + f_3(r/t)\,\mathbf{k} \quad (r = \sqrt{x^2+y^2}) \tag{1.2}$$

where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors directed along the x, y, z axes respectively. We can therefore represent the dislocation crack as the sum of shear, shift and antiplane shear cracks and obtain the following boundary conditions ($\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{xz}, \sigma_{yz}$ are the components of the stress tensor):

$$\begin{aligned} &U_x = {}^1/_2 f_1(r/t), \ \sigma_{yy} = 0 \ \text{ when } \ y = 0, \ 0 < x < vt \\ &U_x = 0, \ \sigma_{yy} = 0 \ \text{ when } \ y = 0, \ x < 0, \ x > vt \end{aligned} \tag{1.3}$$

for the shear discontinuity (the problem is skew symmetric with respect to the x axis),

$$\begin{aligned} &U_y = {}^1/_2 f_2(r/t), \ \sigma_{xy} = 0 \ \text{ when } \ y = 0, \ 0 < x < vt \\ &U_y = 0, \ \sigma_{xy} = 0 \ \text{ when } \ y = 0, \ x < 0, \ x > vt \end{aligned} \tag{1.4}$$

for the shift discontinuity (the problem is symmetric with respect to the x axis),

$$\begin{aligned} &U_z = {}^1/_2 f_3(r/t) \ \text{ when } \ y = 0, \ 0 < x < vt \\ &U_z = 0 \ \text{ when } \ y = 0, \ x < 0, \ x > vt \end{aligned} \tag{1.5}$$

for the antiplane shear discontinuity.

The problems formulated above are selfsimilar. We shall solve these problems using the general approach /24/ based on representing the solutions of the equations in terms of analytic functions of a complex variable, enabling us to formulate the selfsimilar problems as certain Riemann-Hilbert problems for a half-plane.

If the functions LU_x, LU_y, LU_z and homogeneous, then the following notation can be used:

$$\begin{aligned} &U_x^\circ = LU_x, \quad U_y^\circ = LU_y, \quad U_z^\circ = LU_z, \quad \overset{\circ}{\sigma}_{xx} = L\sigma_{xx}, \quad \overset{\circ}{\sigma}_{yy} = L\sigma_{yy} \\ &\overset{\circ}{\sigma}_{xy} = L\sigma_{xy}, \quad \overset{\circ}{\sigma}_{xz} = L\sigma_{xz}, \quad \overset{\circ}{\sigma}_{yz} = L\overset{\circ}{\sigma}_{yz}; \quad L = \partial^{m+n}/\partial x^m \partial t^n \end{aligned} \tag{1.6}$$

Then the general representation of the solutions in terms of a single analytic function of complex variables

$$z_k = \frac{tx - iy\sqrt{t^2 - c_k^{-2}(x^2 + y^2)}}{x^2 + y^2} \quad (k = p, s)$$

will have the form

$$\begin{aligned} &U_x^\circ = \operatorname{Re}[U_p(z_p) + U_s(z_s)], \quad U_y^\circ = \operatorname{Re}[V_p(z_p) + V_s(z_s)], \quad U_z^\circ = 0 \\ &\overset{\circ}{\sigma}_{xx} = \frac{\mu}{c_s^{-2}} \operatorname{Re}\left\{\frac{[c_s^{-2} - 2(c_p^{-2} - z_p^2)](c_p^{-2} - 2z_p^2)}{c_p^{-2} - z_p^2} W_2'(z_p)\frac{\partial z_p}{\partial y} - \right. \\ &\qquad \left. 4z_p^2 W_2'(z_s)\frac{\partial z_s}{\partial y}\right\} \\ &\overset{\circ}{\sigma}_{yy} = \frac{\mu}{c_s^{-2}} \operatorname{Re}\left\{\frac{(c_s^{-2} - 2z_p^2)^2}{c_p^{-2} - z_p^2} W_2'(z_p)\frac{\partial z_p}{\partial y} + 4z_s^2 W_2'(z_s)\frac{\partial z_s}{\partial y}\right\} \\ &\overset{\circ}{\sigma}_{xy} = \frac{2\mu}{c_s^{-2}} \operatorname{Re}\left\{(c_s^{-2} - 2z_p^2) W_2'(z_p)\frac{\partial z_p}{\partial x} - (c_s^{-2} - 2z_s^2) W_2'(z_s)\frac{\partial z_s}{\partial x}\right\}. \\ &U_p'(z) = \frac{z(c_s^{-2} - 2z^2)}{c_s^{-2}\sqrt{c_p^{-2} - z^2}} W_2'(z), \quad V_p'(z) = \frac{c_s^{-2} - 2z^2}{c_s^{-2}} W_2'(z) \\ &U_s'(z) = -\frac{2z\sqrt{c_s^{-2} - z^2}}{c_s^{-2}} W_2'(z), \quad V_s'(z) = \frac{2z^2}{c_s^{-2}} W_2'(z) \end{aligned} \tag{1.7}$$

for problems symmetrical about the x axis /24/,

$$\begin{aligned} &U_x^\circ = \operatorname{Re}[U_p(z_p) + U_s(z_s)], \quad U_y^\circ = \operatorname{Re}[V_p(z_p) + V_s(z_s)], \quad U_z^\circ = 0 \\ &\overset{\circ}{\sigma}_{xx} = \frac{2\mu}{c_s^{-2}} \operatorname{Re}\left\{[c_s^{-2} - 2(c_p^{-2} - z_p^2)] W_1'(z_p)\frac{\partial z_p}{\partial x} + \right. \\ &\qquad \left. (c_s^{-2} - 2z_s^2) W_1'(z_s)\frac{\partial z_s}{\partial x}\right\} \\ &\overset{\circ}{\sigma}_{yy} = \frac{2\mu}{c_s^{-2}} \operatorname{Re}\left\{(c_s^{-2} - 2z_p^2) W_1'(z_p)\frac{\partial z_p}{\partial x} - (c_s^{-2} - 2z_s^2) W_1'(z_s)\frac{\partial z_s}{\partial x}\right\} \\ &\overset{\circ}{\sigma}_{xy} = \frac{\mu}{c_s^{-2}} \operatorname{Re}\left\{4z_p\sqrt{c_p^{-2} - z_p^2}\, W_1'(z_p)\frac{\partial z_p}{\partial x} + \frac{(c_s^{-2} - 2z_s^2)^2}{z_s\sqrt{c_s^{-2} - z_s^2}} W_1'(z_s)\frac{\partial z_s}{\partial x}\right\} \end{aligned} \tag{1.8}$$

$$U_p'(z)=\frac{2z^2}{c_s^{-2}}W_1'(z),\quad V_p'(z)=\frac{2z\sqrt{c_j^{-2}-z^2}}{c_s^{-2}}W_1'(z)$$
$$U_s'(z)=\frac{c_s^{-2}-2z^2}{c_s^{-2}}W_1'(z),\quad V_s'(z)=-\frac{z(c_s^{-2}-2z^2)}{c_s^{-2}\sqrt{c_s^{-2}-z^2}}W_1'(z)$$

for the problems skew symmetric about the x axis /9, 22/, and

$$U_x^\circ=U_y^\circ=0,\quad U_z^\circ=\operatorname{Re}W_3(z_s),\quad \overset{\circ}{\sigma}_{xz}=\mu\operatorname{Re}W_3'(z_s)\frac{\partial z_s}{\partial x},\qquad (1.9)$$
$$\overset{\circ}{\sigma}_{yz}=\mu\operatorname{Re}W_3'(z_s)\frac{\partial z_s}{\partial y}$$

for the antiplane case.

Using the representations (1.7)-(1.9), we formulate the boundary value problem (1.1)-(1.5) in the form of the following Dirichlet problem:

$$\operatorname{Re}W_j(z)={}^1/_2 f_j(1/z)\ \text{when}\ \operatorname{Im}z=0,\ \operatorname{Re}z>v^{-1}\qquad (1.10)$$
$$\operatorname{Re}W_j(z)=0\ \text{when}\ \operatorname{Im}z=0,\ \operatorname{Re}z<v^{-1}$$

The solution of the boundary value problem (1.10) is given by the Schwarz integral /25/

$$W_j\left(\frac{1}{z}\right)=\frac{1}{2\pi i}\int_0^v\frac{f_j(t)\,dt}{t-z}+iC_0\qquad (1.11)$$

Knowing the function $W_j(z)$, we can use formulas (1.7)-(1.9) to write the stress and displacement components, remembering that $L=1$ since the displacements are uniform functions.

Let us consider in more detail the case when

$$\mathbf{f}(r/t)=\mathbf{B}(b_1,\ b_2,\ b_3)=\text{const}$$

In this case the solution of (1.10) has the form

$$W_j(z)=-\frac{ib_j}{2\pi}\ln(1-vz),\quad i^2=-1\qquad (1.12)$$

Substituting (1.12) into (1.7)-(1.9) and summing the solutions obtained, we obtain the following expressions for the displacement field:

$$U_x=U_x^p+U_x^s,\quad U_y=U_y^p+U_y^s,\quad U_z=\frac{b_3}{2\pi}\operatorname{arc}_s\qquad (1.13)$$
$$U_x^p=Ab_1[2\operatorname{arc}_p-F_p]-Ab_2[(\beta_3^2-\gamma^2)\ln_p+{}^1/_2\beta_3^2\beta_1^{-1}\operatorname{Ln}_p-f_p]$$
$$U_x^s=Ab_1[\beta_3^2\operatorname{arc}_s+F_s]+Ab_2[\beta_3^2\ln_s-\beta_3\operatorname{Ln}_s-f_s]$$
$$U_y^p=Ab_1[(1+\beta_1^2)\ln_p+\beta_1\operatorname{Ln}_p+f_p]+Ab_2[\beta_3^2\operatorname{arc}_p+F_p]$$
$$U_y^s=-Ab_1[2\ln_s-{}^1/_2\beta_3^2\beta_2^{-1}\operatorname{Ln}_s+f_s]+Ab_2[2\operatorname{arc}_s-F_s]$$
$$\operatorname{Ln}_p=\ln\frac{(n_p\beta_1-m_p)^2\sin^2\varphi+[\gamma-(n_p-m_p\beta_1)\cos\varphi]^2}{(1-\gamma n_p\cos\varphi)^2+\gamma^2m_p^2\sin^2\varphi}$$
$$\operatorname{Ln}_s=\ln\frac{(n_s\beta_2-m_s)^2\beta^2\sin^2\varphi+[\gamma-(n_s-m_s\beta_2)\beta\cos\varphi]^2}{(\beta-\gamma n_s\cos\varphi)^2+\gamma^2m_s^2\sin^2\varphi}$$
$$\ln_p=\ln(n_p+m_p),\quad \ln_s=\ln(n_s+m_s)$$
$$\operatorname{arc}_p=\operatorname{arctg}\frac{\gamma m_p\sin\varphi}{1-\gamma n_p\cos\varphi},\quad \operatorname{arc}_s=\operatorname{arctg}\frac{\gamma m_s\sin\varphi}{\beta-\gamma n_s\cos\varphi}$$
$$F_p=\gamma m_p(\gamma n_p\sin2\varphi+2\sin\varphi),\quad f_p=\gamma m_p(\gamma n_p\cos2\varphi+2\cos\varphi)$$
$$F_s=\gamma\beta^{-1}m_s(\gamma\beta^{-1}n_s\sin2\varphi+2\sin\varphi),$$
$$f_s=\gamma\beta^{-1}m_s(\gamma\beta^{-1}n_s\cos2\varphi+2\cos\varphi)$$
$$A=\frac{\beta^2}{2\pi\gamma^2},\quad \beta_1^2=1-\gamma^2,\quad \beta_2^2=1-\frac{\gamma^2}{\beta^2},\quad \beta_3^2=\frac{\gamma^2}{\beta^2}-2,$$
$$\beta=\frac{c_s}{c_p},\quad \gamma=\frac{v}{c_p}$$
$$n_k=\frac{tc_k}{r},\quad m_k=\begin{cases}\sqrt{n_k^2-1}, & n_k>1\\ 0, & n_k<1\end{cases}\ (k=p,s);\quad \begin{aligned}r&=\sqrt{x^2+y^2}\\ \varphi&=\operatorname{arctg}y/x\end{aligned}$$

In a small neighbourhood of the moving crack edge, i.e. when $r_1=\sqrt{(x-vt)^2+y^2}\to0$ and $\varphi_1=\operatorname{arctg}[y/(x-vt)]$, the stress field behaves as follows:

$$\sigma_{rr}=\sigma_{xx}\cos^2\varphi_1+\sigma_{yy}\sin^2\varphi_1+\sigma_{xy}\sin2\varphi_1=[b_1K_{rr}^{\mathrm{II}}+b_2K_{rr}^{\mathrm{I}}]D\qquad (1.14)$$
$$\sigma_{\varphi\varphi}=\sigma_{xx}\sin^2\varphi_1+\sigma_{yy}\cos^2\varphi_1-\sigma_{xy}\sin2\varphi_1=[b_1K_{\varphi\varphi}^{\mathrm{II}}+b_2K_{\varphi\varphi}^{\mathrm{I}}]D$$

$$\sigma_{r\varphi} = (\sigma_{yy} - \sigma_{xx}) \sin\varphi_1 \cos\varphi_1 + \sigma_{xy} \cos 2\varphi_1 = [b_1 K^{\mathrm{II}}_{r\varphi} + b_2 K^{\mathrm{I}}_{r\varphi}] D$$
$$D^{-1}\gamma^2\beta^{-2}\sigma_{xx} = -2b_1 \{\gamma_1^{-1}\beta_1 [\gamma^2 + 2\beta^2\beta_1^2] \beta^{-2} + \gamma_2^{-1}\beta_3^2\beta_2\} \times$$
$$\sin\varphi_1 - b_2 \{\beta_3^2\beta^{-2}\gamma_1^{-1}\beta_1^{-1} [\gamma^2 + 2\beta^2\beta_1^2] + 4\beta_2\gamma_2^{-1}\} \cos\varphi_1$$
$$D^{-1}\gamma^2\beta^{-2}\sigma_{yy} = -2b_1\beta_3^2 (\beta_1\gamma_1^{-1} - \beta_2\gamma_2^{-1}) \sin\varphi_1 -$$
$$b_2 (\beta_3^4\gamma_1^{-1}\beta_1^{-1} - 4\beta_2\gamma_2^{-1}) \cos\varphi_1$$
$$D^{-1}\gamma^2\beta^{-2}\sigma_{xy} = b_1 (4\beta_1\gamma_1^{-1} - \beta_3^4\beta_2^{-1}\gamma_2^{-1}) \cos\varphi_1 -$$
$$2b_2\beta_3^2 (\beta_1\gamma_1^{-1} - \beta_2\gamma_2^{-1}) \sin\varphi_1$$
$$\sigma_{xz} = -Db_3\beta_2\gamma_2^{-1} \sin\varphi_1, \quad \sigma_{yz} = Db_3\beta_2\gamma_2^{-1} \cos\varphi_1$$
$$\gamma_1 = (1 - \gamma^2 \sin^2\varphi_1), \quad \gamma_2 = (1 - \gamma^2\beta^{-2} \sin^2\varphi_1), \quad D = \mu/2\pi r_1$$

The functions (1.13) and (1.14) play the part of the fundamental solutions, which can be used, with help of the principle of superposition, to construct the solution of the problem for an arbitrary system of cracks propagating with arbitrary variable velocities along arbitrary curvilinear trajectories.

2. Construction of the general solution. Suppose we have m principal directions in which the curvilinear cracks begin to propagate at instants of time $t = t_0^\delta$ ($\delta = 1, 2, \ldots, m$) from the points (x_0^δ, y_0^δ). We shall assume that the trajectory of every crack consists of n^δ rectilinear segments making angles α_i^δ ($i = 0, 1, \ldots, n-1$) with the x axis, the tips of these segments moving with velocities v_{i+1}^δ. Let the values of the displacement vector components at every rectilinear segment of the crack be $b_{j,\,i+1}^\delta$. We will assume that the instants of time t_i^δ correspond either to the instants of change of direction, or to the instants at which the rectilinear cracks stop.

Then the solution of this problem will consist of the superposition of the fundamental soltuions (1.13) constructed above, with the obvious change of arguments

$$x \to \bar{x}_i, \quad y \to \bar{y}_i, \quad t \to t - t_i^\delta, \quad v \to v_{i+1}^\delta, \quad b_j \to b_{j,\,i+1}^\delta \tag{2.1}$$

and will have the following form:

$$U = \sum_{\delta=1}^{m} \Big[\sum_{i=0}^{n-1} U^\delta (b_{j,\,i+1}^\delta;\ v_{i+1}^\delta;\ t - t_{2i}^\delta;\ \bar{x}_{2i}^\delta;\ \bar{y}_{2i}^\delta) - \tag{2.2}$$
$$\sum_{i=0}^{n-1} U^\delta (b_{j,\,i+1}^\delta;\ v_{i+1}^\delta;\ t - t_{2i+1}^\delta;\ \bar{x}_{2i+1}^\delta;\ \bar{y}_{2i+1}^\delta) \Big]$$

$$\bar{x}_{2i}^\delta = (x - x_i^\delta) \cos\alpha_i^\delta + (y - y_i^\delta) \sin\alpha_i^\delta \tag{2.3}$$
$$\bar{x}_{2i-1}^\delta = (x - x_i^\delta) \cos\alpha_{i-1}^\delta + (y - y_i^\delta) \sin\alpha_{i-1}^\delta$$
$$\bar{y}_{2i}^\delta = -(x - x_i^\delta) \sin\alpha_i^\delta + (y - y_i^\delta) \cos\alpha_i^\delta$$
$$\bar{y}_{2i-1}^\delta = -(x - x_i^\delta) \sin\alpha_{i-1}^\delta + (y - y_i^\delta) \cos\alpha_{i-1}^\delta$$
$$x_i^\delta = x_0^\delta + \sum_{l=1}^{i} v_l^\delta (t_{2l-1} - t_{2l-2}) \cos\alpha_{l-1}^\delta$$
$$y_i^\delta = y_0^\delta + \sum_{l=1}^{i} v_l^\delta (t_{2l-1} - t_{2l-2}) \sin\alpha_{l-1}^\delta$$

Here the upper and lower indices on the displacements U are the same as in the fundamental solution given by (1.13), and are therefore omitted in order to simplify the expressions.

We shall assume that the trajectories of curvilinear cracks are given for every principal direction in the form of smooth functions $g^\delta(x, y, z, t)$. Let the values of the displacement vectors on every separate curvilinear crack be given in the form of uniquely specified functions $b^\delta(x, y, z, t)$, and the velocities of propagation of the crack edges in the form of the functions $v^\delta(t)$. Then the theoretical seismograms can be found using the following well-known procedure.

By decomposing every curvilinear trajectory into a series of piecewise-rectilinear segments and determining the mean values of the functions $b_i^\delta(x, y, z, t)$ and $v_i^\delta(t)$ for every segment, we can use the solutions (1.13) and (2.2). According to this solution, the displacement jump and the velocities of motion of the crack edges will be constant on every piecewise linear segment. Then, by increasing the number of segments we can bring the solution (1.13), (2.2) as close to the solution of the problem of the propagation of the cracks with variable velocities along the smooth curvilinear trajectories as we please, since it is obvious that if we pass to the limit in the solution (1.13), (2.2), (2.3) as

$$n \to \infty, \quad t_{i+1}^\delta - t_i^\delta \to 0, \quad v_{i+1}^\delta - v_i^\delta \to 0 \tag{2.4}$$
$$b_{j,\,i+1}^\delta - b_{j,\,i}^\delta \to 0, \quad \alpha_{i+1}^\delta - \alpha_i^\delta \to 0$$

then the solution of theproblem with given piecewise-linear functions g_i^δ, b_i^δ v_i^δ can be brought as close as desired to the solution of the problem of the propagation of cracks with variable functions g^δ, b^δ, v^δ.

Note that the approach proposed here can be especially effective in analysing seismograms and can serve as a means of establishing new relations between the parameters of the theoretical seismograms and of complex cracks occurring in the focus zones of tectonic earthquakes.

3. Analysis of the solution and numerical calculations. The study of how the stresses (1.14) near the moving crack edge depend on the angle φ_1 measured from the direction in which the crack propagates, and on its velocity of propagation, show that beginning from some value of the critical velocity v_*, two symmetrical maxima in the stresses $\sigma_{r\varphi}$ in the case of pure shear discontinuity and in the stresses $\sigma_{\varphi\varphi}$ in the case of pure dislocation discontinuity appear near $\varphi_1 = 0$. This implies that the conditions for the rectilinear propagation of the crack become unstable as the velocity increases. Thus, when the velocity of motion of the crack tip continues to increase further, i.e. when $v > v_*$, rectilinear crack propagation becomes impossible since either the trajectory will curve, or the crack will begin to branch. The corresponding equation for this critical velocity is the same for both the shear and the dislocation discontinuity /17, 19/. The equation is found from the condition that

$$\left.\frac{\partial^2 \sigma_{r\varphi}}{\partial \varphi_1^2}\right|_{\varphi_1=0} = 0 \quad \text{for pure shear;} \quad \left.\frac{\partial^2 \sigma_{\varphi\varphi}}{\partial \varphi_1^2}\right|_{\varphi_1=0} = 0 \quad \text{for pure dislocation} \tag{3.1}$$

and has the form /17/

$$4\sqrt{1-\alpha^2\beta^2}\sqrt{1-\alpha^2} - (\alpha^2-2)^2 - 2(2-\alpha^2)\alpha^4 = 0, \quad v_* = \alpha c_s \tag{3.2}$$

Let us give some values for the root α of (3.2) depending on the value of Poisson's ratio ν

α	0.46	0.485	0.515	0.55	0.59	0.64
ν	0	0.1	0.2	0.3	0.4	0.5

This shows that the critical velocity of rectilinear propagation of dislocation discontinuities of the first type, i.e. of discontinuities with a stress singularity of the type $\sigma_{ij} \approx K_{ij} r^{-1}$ /21/ at the tip of the crack, will be approximately 10% less than that for cracks /26/, i.e. for dislocation discontinuities with a singularity of the type $\sigma_{ij} \approx K_{ij} r^{-1/2}$ at the tip of the crack. We note that in the case of antiplane shear discontinuities the velocity of the shear wave is the limiting velocity.

Fig.1 shows the variation in the stress intensity coefficients K_{rr}^I, $K_{\varphi\varphi}^I$, $K_{r\varphi}^I$ for a pure dislocation discontinuity $b_1 = b_3 = 0$, $b_2 = 1$ (Fig.1a), K_{rr}^{II}, $K_{\varphi\varphi}^{II}$, $K_{r\varphi}^{II}$ for the pure shear discontinuity $b_1 = 1$, $b_2 = b_3 = 0$ (Fig.1b) and K_{rr}^{I+II}, $K_{\varphi\varphi}^{I+II}$, $K_{r\varphi}^{I+II}$ for the complex discontinuity $b_1 = 1$, $b_2 = 1$, $b_3 = 0$ (Fig.1c). The curves *1, 2, 3* correspond to the velocities of propagation of the discontinuities $0.1c_s$, $0.7c_s$, $0.8c_s$. The qualitative behaviour of the graph for $K_{\varphi\varphi}^I$ is the same as that given by Yoffe in /26/.

Fig.1 shows that the increase in velocity is accompanied by an increase in the values of the coefficients K_{rr} in all directions, for all three types of discontinuities; although characteristic maxima appear in the values of the coefficients $K_{\varphi\varphi}$ (with increasing velocity of propagation of the discontinuity), their magnitude decreases in all directions for all types of discontinuity. The behaviour of the coefficients $K_{r\varphi}$ depends essentially on the direction. Thus all three types of discontinuity are characterized by two directions in which the values of $K_{r\varphi}$ are practically independent of the velocity of propagation of the discontinuity. The directions divide the space between the front of the moving discontinuity into two zones. In one of these zones the values of $K_{r\varphi}$ decreases, and in the other it increases.

Thus, when $-63^\circ \leqslant \varphi_1 \leqslant +63^\circ$ for pure dislocation, $-37^\circ \leqslant \varphi_1 \leqslant +37^\circ$ for pure shear and $-23^\circ \leqslant \varphi_1 \leqslant +47^\circ$ for a complex discontinuity, the value of $K_{r\varphi}$ decreases, while for the remaining values of the angle φ_1 it increases with increasing velocity of propagation of the discontinuity. If we then assume that the directions in which the values of the coefficients $K_{r\varphi}$ are practically independent of the velocity of propagation of the discontinuity are responsible for the formation of branched segments of the crack, we obtain the values of the smallest possible branching angles $\varphi_1 \approx \pm 63^\circ$ for a pure disclocation discontinuity, and $\varphi_1 \approx \pm 37^\circ$ for the pure shear discontinuity, and for a complex discontinuity the branching angles will depend on the ratio of the shear and dislocation components of the displacement vector at the discontinuity.

Thus, analysing Fig.1 we find that if theprocess of fundamental development of the discontinuity begins with a pure dislocation element of the fracture, then as the velocity of propagation of the crack increases we have, apart from the appearance of two symmetrical maxima in the stresses $\sigma_{\varphi\varphi}$, also a sharp increases in the stress $\sigma_{r\varphi}$ along specified parts of the zone situated in front of the moving crack edge. This must lead to symmetrical branching of the crack and the formation of considerable shear components of the displacement vector on the bounded segments.

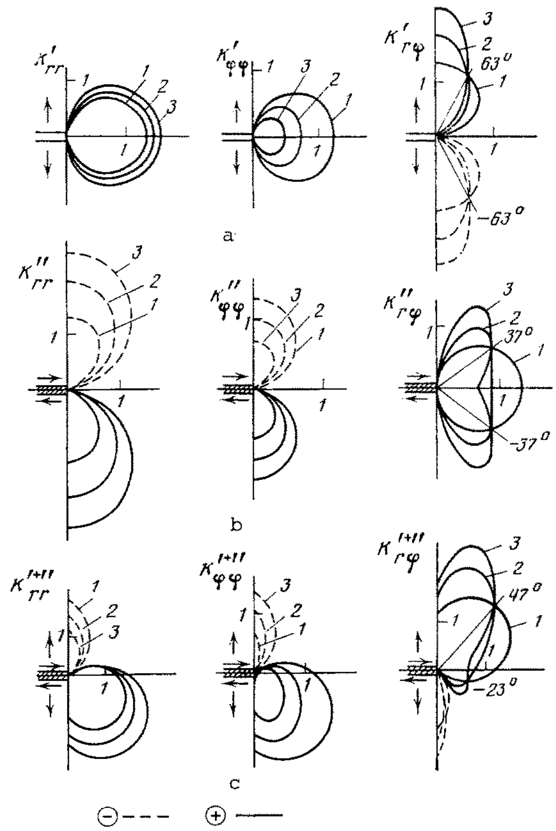

Fig.1

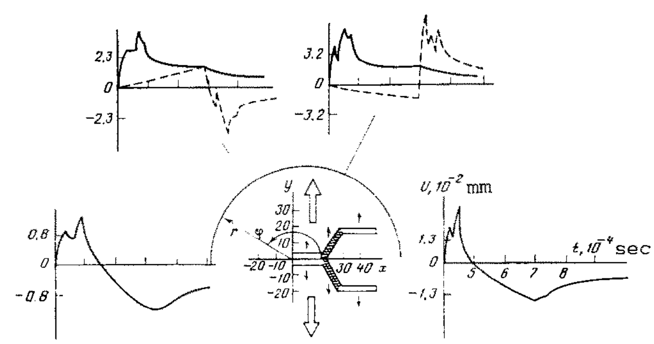

Fig.2

If the fundamental crack propagation begins from the pure shear fracture element, then as the velocity of propgation increases we have, in addition to the appearance of two symmetrical maxima in the stresses $\sigma_{r\varphi}$, also the formation of a discontinuity in the field of tensile stresses on one hand, and of a zone of compressive stresses on the other hand. This should lead either to deviation of the trajectory of motion by an angle φ from its initial direction and the formation of a considerable dislocation component of the displacement vector on the deviating segment of the crack, or to splitting of the crack into two branches. On one of these branches the displacement vector will have only the shear component, and both the shear and the dislocation component on the other branch.

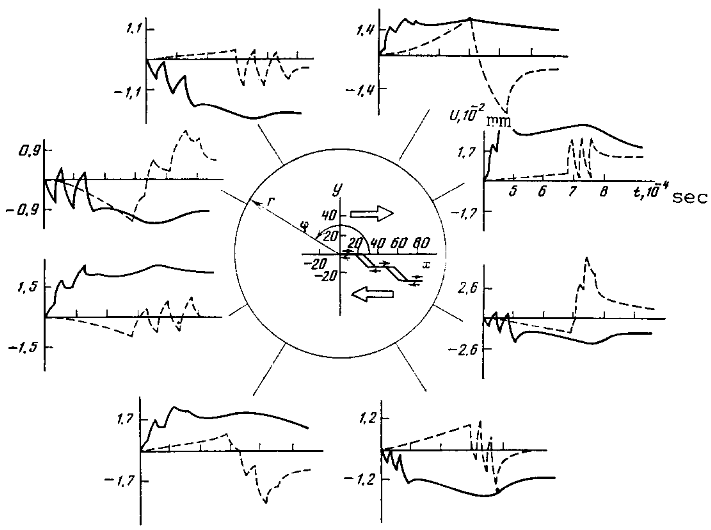

Fig.3

Figs.2 and 3 show theoretical seismograms (displacement-time relations) for the crack branching and bending schemes described above. The solid line depicts the behaviour of the component u_r and the dashed line the component u_φ of the displacement vector. The theoretical seismograms were computed using formulas (1.13), (2.1)-(2.3) with help of a FORTRAN program written to produce the numerical results in the form of graphs. The maximum computer time for a single version including computing three components of the theoretical seismograms and their spectra at 12 observation points and of the diagrams showing the directions of seismic emission, did not exceed 5 min. The initial parameters of the medium and the cracks were as follows: $c_p = 2300$ m/sec, $c_s = 1300$ m/sec, $r = 900$ mm, $m = 2$, $n^1 = n^2 = 3$, $v_i^1 = v_i^2 = 780$ m/sec, $x_0^1 = x_0^2 = y_0^1 = y_0^2 = 0$, $\alpha_0^1 = \alpha_0^2 = 0$, $\alpha_1^1 = -45°$, $\alpha_1^2 = 45°$, $\alpha_2^1 = \alpha_2^2 = 0$, $b_{1,1}^1 = b_{1,1}^2 = 0$, $b_{1,2}^1 = b_{1,2}^2 = 0{,}7$, $b_{1,3}^1 = b_{1,3}^2 = 0$, $b_{2,1}^1 = b_{2,1}^2 = 1$, $b_{2,2}^1 = b_{2,2}^2 = 0{,}7$, $b_{2,3}^1 = b_{2,3}^2 = 1$, $t_0^1 = t_0^2 = 0$, $t_1^1 = t_1^2 = t_2^1 = t_2^2 = 25\cdot10^{-6}$ sec, $t_3^1 = t_3^2 = t_4^1 = t_4^2 = 50\cdot10^{-6}$ sec $t_5^1 = t_5^2 = 75\cdot10^{-6}$ sec for the crack beginning to propagate from a pure dislocation element (Fig.2), and the seismograms were computed for $\varphi = 0, 60, 120$ and $180°$. The parameters $m = 1$, $n^1 = 5$, $v_i^1 = 780$ m/sec, $x_0^1 = y_0^1 = \alpha_0^1 = 0$, $\alpha_1^1 = -45°$, $\alpha_2^1 = 0$, $\alpha_3^1 = -45°$, $\alpha_4^1 = 0$, $t_0^1 = 0$, $t_1^1 = t_2^1 = 25\cdot10^{-6}$ sec, $t_3^1 = t_4^1 = 50\cdot10^{-6}$ sec, $t_5^1 = t_6^1 = 75\cdot10^{-6}$ sec, $t_7^1 = t_8^1 = 100\cdot10^{-6}$ sec and $t_9^1 = 125\cdot10^{-6}$ sec were used for a crack beginning to propagate from a pure shear element (Fig.3) and the seismograms were computed for $\varphi = 30, 60, 120, 150, 210, 240, 300$ and $330°$.

We see that in both cases the form of the theoretical seismograms appears to be considerably complicated by the high-frequency emission manifesting itself in the seismograms by the appearance of characteristic extrema and step-like segments. The segments correspond to the instances of arrival of the waves from the shear and dislocation elements of the crack appearing when the trajectory of motion of the crack becomes curved, and are also connected with the jump-like change in the velocity of propagation. When the path of crack propagation is curvilinear (Fig.3), the seismograms with sign-alternating trains of impulses are observed in separate directions. The sign-alternating train of impulses in the *P*-waves (longitudinal waves) will be sharpest along the principal direction of crack propagation, i.e. it will depend essentially on the position of the seismic focus and the observer, and on the magnitude of the angles of rotation of the separate segments of the crack.

We note that the larger the angles of rotation of the separate crack elements, the more there will be observation points at which the sign-alternating seismograms will be fixed. Thus when the angles of rotation of the separate segments of pure shear cracks are close to $90°$, the sign-alternating signals in *P*-waves will be observed in almost all directions. Analysing Figs.2 and 3 we can see that the duration of the general signal in the direction of motion of the crack is much shorter, i.e. the high frequencies will be higher in the direction of motion than in the opposite direction. This is an obvious example of the Doppler effect. This special feature of the wave field can be used as an important factor in choosing the plane of main propagation of the crack and in determining its direction of propagation.

The integral estimate of the components u_r shows that the total emission in the *P*-waves (over all observed points) will be positive (i.e. compressive), since the principal crack contains mixed elements on which we have the dislocation component of the displacement vector. Therefore, if an analogous effect is discovered when analysing full scale seismograms, we can assert that in the case of the real earthquakes the complex elements of the crack will grow, i.e.

we shall have both the shear as well as the dislocation components of the displacement vector on the principal fracture surface.

Thus, analysing in more detail the recordings of full-scale seismograms over a number of stations, we can obtain additional information concerning the character of the fracture process occurring within the earthquake foci.

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